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REDUCING THE EQUATIONS OF MOTION OF A VISCOUS GAS TO QUADRATURES

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1. There is a certain case for a viscous incompressible liquid (a current in a confuser [1]) when the equations of motion are reduced to quadratures. In the present work, we show that the much more complicated problem of the integration of the equations of motion of a viscous compressible gas for definite class parameters may also be reduced to quadratures. Moreover, we can reduce the equations of the boundary layer of gas to a very simple form (no more complicated than the corresponding equations for a liquid).

The equations of the boundary layer of a compressible gas are:

$$\frac{\partial \psi}{\partial y}\frac{\partial u}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial u}{\partial y} = -\frac{dp}{dx} + \frac{\partial \tau}{\partial y}; \tag{1}$$

$$\rho u = \frac{\partial \psi}{\partial y}, \quad \rho v = -\frac{\partial \psi}{\partial x}; \tag{2}$$

$$p = R\rho T; \tag{3}$$

$$\frac{\mu}{\mu_0} = \left(\frac{T}{T_0}\right)^n. \tag{4}$$

Here n is the experimental constant; the index 0 corresponds to the boundary point.

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In a manner similar to that in our other reports [2, 3], we will interpolate the independent variables

$$x \text{ and } Y = \frac{u}{\sqrt{2c_pT_0}} .$$

Likewise, we will interpolate the dependent variable

$$Z = \frac{1}{\sqrt{2\mu_0\sqrt{2c_pT_0}}}\, p^{-\left[\frac{n(\varkappa-1)}{\varkappa}\right]+1}\, \tau, \qquad (5)$$

where $x = c_p/c_v$ is the index of the adiabatic curve.

Equations (1) and (2) with new independent variables are written as:

$$\frac{\partial\psi}{\partial x} = \frac{dp}{dx}\frac{\mu}{\tau} + \frac{1}{\sqrt{2c_pT_0}}\frac{\partial\tau}{\partial Y};$$

$$\frac{\partial\psi}{\partial Y} = \frac{2c_pT_0\mu\rho Y}{\tau} .$$

Eliminating the unknown function ψ from the equations obtained, using Formulas (3) and (4) together with the integral of energy, and also expressing τ by z according to Formula (5), we finally obtain the equation:

$$z^2\frac{\partial^2 z}{\partial Y^2} - mP'(1-Y^2)^n\frac{\partial z}{\partial Y} - PY(1-Y^2)^{n-1}\frac{\partial z}{\partial x} = 0. \quad (6)$$

Here

$$P = \frac{\varkappa}{\varkappa-1}p^{-\frac{\varkappa-1}{2\varkappa m}} = \frac{\varkappa}{\varkappa-1}\left(1+\frac{\varkappa-1}{2}B^2\right)^{\frac{1}{2m}},$$

where B is Barstou-Mach's number: $m = \frac{\varkappa-1}{2[(2n+1)\varkappa-2n]} = 0.100$

(since for air, $x = 1.4$ and $n = 0.76$).

2. First of all, we shall study the class of internal solutions determined by Fourier's substitution $z = A(x)\,B_n(Y)$.

Assuming

$$\frac{mP'}{A^2} = -\alpha, \qquad (*)$$

$$\frac{P}{2}\left(\frac{1}{A^2}\right)' = 2n\alpha, \qquad (**)$$

where α is a certain constant, we obtain the equation for the determination of $B_n(Y)$:

$$B_n^2\frac{d^2B_n}{dY^2} + \alpha(1-Y^2)^n\frac{dB_n}{dY} + 2n\alpha Y(1-Y^2)^{n-1}B_n = 0$$

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This equation is integrated in quadratures. The first quadrature gives

$$\frac{1}{\alpha}\frac{dB_n}{dY} = \frac{(1-Y^2)^n}{B_n} + constant$$

The arbitrary constant is equal to zero in accordance with the condition at the boundary point.

Performing the second integration, we find that the solution in the vicinity of the boundary point is expressed by the formula

$$B_n(Y) = \sqrt{B_{n_0}^2 + 2\alpha\int_0^Y (1-Y^2)^n\, dY.} \qquad (7)$$

3. We will now determine the distribution of pressures to which the case under consideration corresponds. Juxtaposing (*) and (**) and also employing the relationship between P and p, we find that the solution obtained corresponds to the distribution of pressures according to the hyperbolic law

$$p = \frac{1}{ax+b}, \qquad (8)$$

where a and b are certain fixed coefficients.

We should note that the class parameter studied is interesting not only in itself, but also as a possible basis for the consideration of the general theory. Actually, if the rapidly established curve of distribution of pressures is p(x), then, substituting for the graphic curve 1/p(x) a polygon of the form

$$\frac{1}{p(x)} = a_i x + b_i,$$

for each of whose sides it is possible to use the results of the particular solution obtained, we will obtain, by means of Howards' method, an internal solution for the general case of the flow of a nonadiabatic gas.

4. The methods of the present work and those of other works [2, 3] may be generalized also to the cases of flow around rotating bodies. Limiting ourselves for the sake of simplicity to the case of an incompressible liquid, and interpolating the independent variables x and u and the dependent variable

$$Z = \frac{z}{\sqrt{\mu}\, r(x)} \qquad (9)$$

(here r(x) is the equation of the surface of the rotating body), we obtain the following equation for z

$$z^2\frac{\partial^2 z}{\partial u^2} - \frac{1}{r^2(x)}\left(u\frac{\partial z}{\partial x} + \bar{u}\bar{u}'\frac{\partial z}{\partial u}\right) = 0$$

(here ū is the value of u at the boundary layer).

Proceeding further to the independent variables x and $\xi = u(x, y)/\bar{u}(x)$, we obtain the equation

$$z^2\frac{\partial^2 z}{\partial \xi^2} - a(x)(1-\xi^2)\frac{\partial z}{\partial \xi} - b(x)\xi\frac{\partial z}{\partial x} = 0; \qquad (10)$$

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here $a(x) = \bar{u}^2 \bar{u}'/r^2 (x)$; $b(x) = \bar{u}^3/r^2 (x)$.

Without examining the problems of the integration of Equation (10) in general, we will demonstrate that there exists a case when Equation (10) is reduced to quadratures.

Having assumed $z = \bar{u}^2 M(\xi)$, we obtain for the determination of $M(\xi)$ the equation

$$M^2 \frac{d^2 M}{d\xi^2} + k\left[(1-\xi^2)\frac{dM}{d\xi} + 2\xi M\right] = 0, \qquad (11)$$

where the constant k is determined by the formula

$$k = \frac{\bar{u}'}{\bar{u}^2 r}. \qquad (12)$$

The quadrature of Equation (11) gives

$$M(\xi) = \sqrt{2k}\sqrt{\frac{2}{3} - \xi + \frac{\xi^3}{3}}. \qquad (13)$$

From Equation (12) is determined the relation between the distribution of speed potentials and the equation for the rotating surface

$$\bar{u} = \frac{1}{c - k\int r\,dx}. \qquad (14)$$

We should note that, if $r =$ constant, we obtain the already known case of a horizontal confuser.

5. We will demonstrate that for the zone of high speeds it is possible to obtain still another quadrature of Equation (6).

In this zone we have $Y \cong \bar{Y}$ (where $\bar{Y}$ is the value of Y at the boundary layer); interpolating the dimensionless variable

$$Y_1 = Y/\bar{Y},$$

we reduce Equation (6) to the form

$$z^2 \frac{\partial^2 z}{\partial Y_1^2} + (1-\bar{Y}^2)^{n-1}\left\{\left[P\bar{Y}^2 \bar{Y}' Y_1^2 - mP'\bar{Y}(1-Y_1^2)\right]\frac{\partial z}{\partial Y_1} - \right.$$
$$\left. - P\bar{Y}^3 Y_1 \frac{\partial z}{\partial x}\right\} = 0$$

But

$$P = \frac{\varkappa}{\varkappa - 1}(1-\bar{Y}^2)^{-1/2n},$$

and finally the equation is written as

$$z^2 \frac{\partial^2 z}{\partial Y_1^2} + P\bar{Y}^2(1-\bar{Y}^2)^{n-1}\left[\bar{Y}'(Y_1^2-1)\frac{\partial z}{\partial Y_1} - \bar{Y}Y_1\frac{\partial z}{\partial x}\right] = 0 \qquad (15)$$

We will look for a solution in the form of Fourier's $z = Y^2 L(Y_1)$.

Assuming
$$\frac{P(1-\bar{Y}^2)^{n-1}\bar{Y}'}{\bar{Y}^2} = -k,$$

we obtain the equation

$$L^2 \frac{d^2L}{dY_1^2} + k\left[(1-Y_1^2)\frac{dL}{dY_1} + 2Y_1 L\right] = 0, \qquad (16)$$

which coincides exactly with Equation (11), and which is similarly integrated in quadratures:

$$L \frac{dL}{dY_1} = k(1-Y_1^2) + \text{constant}.$$

BIBLIOGRAPHY

1. Pohlhausen, H., ZAMM, I (1921)

2. Faynzil'ber, A. M., DAN, 48, No 7 (1945)

3. Faynzil'ber, A. M., DAN, 48, No 8 (1945)

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